DREAM
Danish Research Institute for
Economic Analysis and Modelling

MAKRO

# Estimating CES production functions in MAKRO

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# Summary

In this working paper, we estimate the elasticities of substitution between the different production factors in MAKRO. We use annual data for the period 1967-2017, primarily based on the national accounts. As the technological development is unobservable and potentially non-linear, a trend process is specified for the relative factor efficiency that is time-varying. We use the Kalman filter to estimate the CES elasticities and the technological development simultaneously. It is assumed that the latter develops sluggishly and expresses long-term trends.

The elasticity between capital and labor in the two primary private industries in MAKRO, manufacturing and services, are estimated at 0.51 and 0.42, respectively, and are both significantly different from 1. The elasticities towards buildings are most often estimated at 0, meaning that buildings are a Leontief input factor. This also applies to some extent to materials, but in the construction and manufacturing industries, where materials make up a large part of the total input in production, these elasticities are estimated at around 0.5.

In the long run, the technological development is labor-augmenting but there are significant periods of shift in the direction of capital-augmenting technology, e.g. in the service industry from the 1990s onwards, highlighting the importance of a flexible specification of technological development. The approach in the analysis provides well-specified models with parameter estimates similar to other analyzes on similar data for Denmark. However, point estimates are obtained with some uncertainty, which is probably a combination of uncertainty behind the trend specification and volatile prices, especially in the user cost terms.

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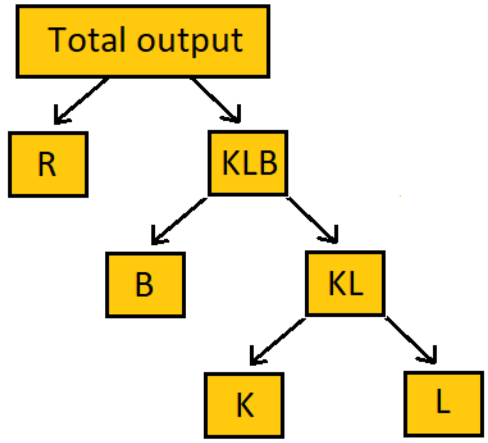
# 1 Introduction

In MAKRO, it is assumed that the production of goods and services is achieved using the following factors as input: machinery capital, labor, buildings and resources, where the latter in MAKRO is a weighting of energy and material inputs (i.e. domestically or foreign produced intermediate products). We use a nested CES production function, where a choice is made between two factors at a time. The nested CES structure allows for varying degrees of substitution between factors for changes in prices and technology. The factor demand is determined partly by the total production that increases the demand for all factors, and partly by substitution between factors resulting from shifts in the relative relationship between the factors' price and efficiency. Technological changes, e.g. labor augmenting technological changes, express shifts in the relative efficiency of the factors resulting in substitution that cannot be explained by a shift in the relative prices. A well-known problem arises from the fact that the last two effects cannot be separated without further assumptions. In other words, estimation requires that one makes an identifying assumption about the development of the technology over time.

We follow the method described in detail in the DREAM working paper Kronborg et al. (2019), which uses the Kalman Filter to estimate CES elasticities and a process of technological development simultaneously. The method is implemented in an R package available on the DREAM group's website. We identify technological development by assuming that it is a sluggish process expressing the long run trends in the economy. Additionally, the adjustment to the desired factor demand is allowed to be sluggish - e.g. due to adjustment costs of capital - and we therefore estimate the factor demand using an error correction model. We look at the following private industries: agriculture, construction, energy, extraction, manufacturing, sea transportation and services. Each of these industries is assumed to use the mentioned input factors of production. We estimate the production functions by applying annual data covering the period 1967-2017. In this working paper, the results of the estimations are presented and they are compared to other studies. In addition, we check the robustness to a number of methodological choices.

The elasticity between capital and labor in the two largest private industries in MAKRO, manufacturing and services, is estimated to be 0.51 and 0.42, respectively. In the other industries, this elasticity is estimated to be approximately 0.1, except for the extraction industry for which it is 0.33. The elasticities between capital and labor combined and buildings are most often estimated to be 0. In some cases, the elasticity between capital, labor, and buildings combined and resources is also 0. The very low elasticities are in line with other Danish literature (e.g. Gustafsson, 2014), and may be due to a downward bias in the estimates due to noisy user cost expressions or a so-called attenuation bias. Investigating the cause behind the low elasticities in more detail is interesting, but for now left to future work. Our results show that the overall technological development has

Figur 1: Nest structure in MAKRO's production function.




*K, L, B and R are machinery capital, labor, buildings and resources, respectively. Nested factors are denoted by multiple letter, for example "KL"is the aggregate of machinery capital and labor input.*

been aimed at improving the efficiency of labor, so-called Harrod-neutral technological growth. However, there are periods of shift in the direction of technology, especially in the 90s in the service industry and after the financial crisis across industries.

The structure in the rest of the working paper is as follows: In Section 2 the nest structure and the estimated model are presented. We also explain some methodological considerations as well as alternative specifications tried previously. Section 3 describes the data used, including the expression for user cost of capital. The estimated elasticities and trends in technology are presented in Section 4. Section 5 contains a robustness analysis, while Section 6 summarizes.

## 2 Description of method

The production follows a nested CES production function, where the firms use machinery capital (K), labor (L), buildings (B) and resources, i.e. energy and material input (R) as factor input. We assume that the firm first chooses between K and L, which combined provide a KL aggregate ("at the bottom" of the nest structure). Secondly, the firm chooses between the aggregate KL and B. Thirdly, the firm chooses between the aggregate KLB and R ("at the top" of the nest structure). In short, we say that production follows a KLBR structure. This structure is illustrated in Figure 1.

The production in each nest is given by

$$Y_t = \left[ (\Gamma_{1t} X_{1t})^{\frac{\sigma-1}{\sigma}} + (\Gamma_{2t} X_{2t})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $Y_t$ is total output or an aggregate component (e.g. the KL aggregate) in a given nest at time $t$, and $X_{it}$ is the quantity of production factor $i$ in the given nest. $\sigma$ is the constant elasticity of substitution, i.e. the percentage change in relative demand between $X_{1t}$ and $X_{2t}$ resulting from a one percentage change in the relative price. $\Gamma_{it}$ is an (unobserved) efficiency index indicating augmenting technological changes for factor$i$. In each nest, the price is formed as a Paasche price index, and the quantity demanded comes from an assumption of cost minimization conditioning on a given nest aggregate. With the assumption of cost minimizing firms, we obtain the well-known CES demand function, which can be rewritten to describe the relative expenditure shares for the two factors of production as follows:

$$\log\left(\frac{P_{1t}X_{1t}}{P_{2t}X_{2t}}\right) = (\sigma-1)\log\left(\frac{\Gamma_{1t}}{\Gamma_{2t}}\right) + (1-\sigma)\log\left(\frac{P_{1t}}{P_{2t}}\right), \tag{2}$$

where $P_{it}$ is the price of factor $i$. The special case $\sigma = 1$ corresponds to a Cobb-Douglas production function with constant relative expense ratios. The second special case, $\sigma = 0$, corresponds to a Leontief production function, where there is no substitution between the factors due to price changes, but solely due to technological changes. When $\sigma < 1$ the two factors are complements as a price increase of one factor (e.g. an increase in wage relative to the user cost) will lead to a larger budget share of this factor. Similarly, relative augmenting technological changes aimed at one factor lead to an increase in the budget share of the other factor (e.g. labor-saving technological advances will increase the budget share of capital). Conversely, when $\sigma > 1$, a price increase of one factor will lead to a decrease in the budget share of this factor, while technological advances will increase the budget share of this factor. This illustrates how the interpretation of technological progress depends on elasticity. Finally, it is worth noting that technological changes of a Hicks-neutral nature (total factor productivity) do not affect the relative level of technology between the two factors, $\Gamma_{1t}/\Gamma_{2t}$, and therefore neither the relative expenditure shares.

One can think of equation (2) as the long run or desired ratio of expenditure shares. To allow for short run dynamics, this long run relationship is embedded in an error correction model. In this way, any adjustment costs are taken into account, which means that the adjustment to the desired distribution of factor input in production is slow. In cases where the residuals are autocorrelated, lagged changes in the relative prices and expenditure shares are added.

Finally, $\mu_t \equiv (\sigma-1)\log\left(\frac{\Gamma_{1t}}{\Gamma_{2t}}\right)$ is defined. The process of the unobserved component, $\mu_t$, is assumed to follow an $I(2)$ process. This assumption ensures that technology is a smooth series and is similar to the assumption made in the HP filter, which allows for medium run variation in technological development (see, e.g. Lemoine et al., 2010 for similar application). The method applied can here therefore be seen as being an extended HP filter. The variance ratio between the model residuals and that of $\Delta\mu_t$ also corresponds to the smoothing parameter in an HP filter (which describes the (inverse) signal-noise ratio), $\lambda$. The special case $\lambda = 0$ means that all short run noise in the relative expenditure shares, which is not due to changes in the relative prices, is attributed to technological progress. Conversely, $\lambda \rightarrow \infty$ results in a linear trend assumption about development of the relative factor efficiency over time. We let the degree of smoothing be determined in the following data driven way: For all industries and nests, the model is estimated with a value for $\lambda$ that varies between 100 and 1,000 with an interval length of 10.[1] Then the value that gives the highest likelihood is chosen, provided that the model is well specified - i.e. no autocorrelation and that the filter is well calibrated, measured from a NIS test.[2] This method also has the advantage that the preferred model per construction will be well specified, i.e. that the residuals are not autocorrelated and that the variance restriction results in a well-specified filter. The specification of the relative technology development constitutes the state equation of the model. The error correction equation, which is a reformulation of (2), constitutes the observation equation. Thus, we write the problem as a linear state-space model, which implies that the Kalman filter can be used to estimate the elasticity at the same time as a description of the time-varying technological changes is obtained. The estimation method is freely available in the statistical software program R (see Kastrup et al., 2021 for a description of the package).[3] In Section 5, we perform a robustness check of the effect of different signal-noise ratios on the elasticity estimate.

# 3 Data

As a starting point, we estimate the model based on data for the longest possible period, i.e. annual data for the period 1967-2017.[4] The variables used are partly from the static calibration of MAKRO, partly national accounts based. Capital is adjusting relatively sluggishly, so we have chosen to use all the available years of data in our preferred model, but we also look at the robustness, when using a shorter period. The production functions are estimated at the same industry level as used in MAKRO, i.e. divided into agriculture, construction, energy, extraction, manufacturing, sea transportation and services. In cases where the division of industries in MAKRO different from the national accounts, the factors are weighted together based on their size in nominal terms. The wages and hours worked divided by industry are taken from ADAM's database instead of MAKRO. This is because it is assumed in MAKRO that there is roughly the same wage across industries and

[1]Another alternative is to estimate $\lambda$ freely with maximum likelihood. However, we find in Kronborg et al. (2019) that it does not necessarily lead to a better fit of data than the grid search and that the resulting model is relatively often mispecified.

[2]Normalized Innovation Squared test (NIS) is a test for filter mispecification. We refer to the mentioned working paper for a more detailed description of this test.

[3]The package can be downloaded via github: github_install(»CKastrup/CESKalman«).

[4]Most of our data series start in 1967, but lack two observations on the tax rate on capital. Additionally, the investment price is included with a lag. We will lose the last observation in 2017 because the calibration of the depreciation rate uses the leaded capital variable.

that the labor force in MAKRO is calculated in efficient units rather than the number of hours. As mentioned, a distinction is made between two types of capital: machinery capital and buildings. Therefore, a so-called *user cost* expression (the total cost of owning and using one unit of capital) is calculated for both types of capital. In MAKRO, user cost is derived from the firms' first-order condition. The estimations in this note are based on a static version of this user cost expression (see Appendix A for details).

The tax depreciation rates are from ADAM's database and the static depreciation rate is calibrated to ensure consistency between the series of capital and investment. To avoid too much noise in the real interest rate, we smooth out the expected rate of price increase in the investment price: Instead of static inflation expectations, we use the HP-filtered inflation rate with 100 as the smoothing parameter in the preferred model. This variable is the only one in the user cost expression that is smoothed. As a robustness check, we also estimate on the basis of adaptive inflation expectations. The average bond yield is used as the interest rate variable in the user cost expression, but the banks' average lending rate was also used as a robustness check. We generally find that the results are robust to different interest rate terms and inflation expectations in the user cost expression. The price indices for each nest are calculated as a Paasche chain price index, which is not very different from the CES price index, given that the production function is CES, as it is a superlative price index.[5] The total quantities produced in each nest follow from a zero-profit assumption. The figures in Appendix B contain the relative quantities as well as the inverse relative prices used in estimation of the preferred specification. The figures thus illustrate the relationship between prices and quantities and both data series have had their means subtracted. If the two series follow on-to-one, then it indicates an elasticity of 1.

# 4 Estimation results

This section presents the results of our preferred model and data specification, while Section 5 looks at the robustness of the results to other specifications. Table 1 provides an overview of the point estimates and the associated standard errors for the CES elasticities in the preferred model specification, broken down by industry and by nest. The Tables 5-11 in Appendix C show the estimated models for each CES nest in greater detail. It is natural to compare the results with similar studies on Danish data, e.g. Thomsen (2015) (Thomsen in the following) and the estimates made for use in the ADAM model (ADAM[6] in the following) and therefore some of them are

[5]The Paasche price index will never be a completely accurate representation of the theoretical price index, but will over time adapt to the new weights. This means that the linked index will never get too far away from the CES price index.

[6]The ADAM group periodically updates its estimates, which are documented in a number of working papers - most recently in Gustafsson (2014). Typically, changes in the point estimates between updates are small, but this

Tabel 1: Overview, estimated CES elasticities (preferred specification).

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| Agriculture | 0.08 | 0.00 | 0.00 |
| | (−0.14;0.33) | (−0.10;0.01) | (−0.12;0.04) |
| Construction | 0.03 | 0.00 | 0.41 |
| | (−0.47;0.42) | (−0.22;0.15) | (0.05;0.95) |
| Energy | 0.04 | 0.00 | 0.10 |
| | (−0.31;0.15) | (−0.25;0.21) | (−0.11;0.18) |
| Extraction | 0.33 | 1.57 | 0.00 |
| | (0.05;0.38) | (0.85;2.12) | (−0.32;0.26) |
| Manufacturing | 0.51 | 0.05 | 0.53 |
| | (0.10;0.70) | (−0.18;0.25) | (0.21;0.69) |
| Sea transport | 0.07 | 0.00 | 0.00 |
| | (−0.15;0.21) | (−0.95;0.33) | (−0.20;0.12) |
| Services | 0.42 | 0.00 | 0.00 |
| | (−0.08;0.84) | (−0.15;0.12) | (−0.29;0.41) |

Note: Displays the estimates in the preferred model specification with IWBZ as the interest rate variable and the period 1967-2017.

compared with these analyzes, which roughly find similar elasticities.[7]

For the KL nest, we start by presenting results for the two major private industries, i.e. manufacturing and services: In both we find moderate elasticities: In our preferred model specification, we find point estimates of 0.51 and 0.42, respectively. As with the other results, the short data series used for estimation contribute to a relatively high uncertainty about the estimates. It should be noted that they are relatively close to - or slightly higher than - those found in other Danish studies: For manufacturing, ADAM finds an elasticity of 0.25 (however 0.65 for foods that MAKRO includes in manufacturing), Thomsen finds approx. 0.4. For services, ADAM finds approx. 0.3 and Thomsen 0.5-0.6. Another paper that estimates the elasticity using Danish data and that allows non-linear technological development via a Box-Cox trend is Muck (2017), which finds an elasticity in the range between 0.3 and 0.7. The elasticities for the industries of agriculture, construction, energy and sea transportation are estimated with relative uncertainty, but are generally found to be low across model specifications. Finally, Kastrup (2019) uses the same method as in this paper, but estimates the KL elasticity for a number of OECD countries and finds a KL elasticity for the total private sector that is around 0.3-0.5 for most economies. The Danish elasticity is estimated at 0.52 and hence relatively close to the estimates in the service and manufacturing industries.

Especially for buildings, it is generally difficult to find significantly positive substitution elasticities, which is a typical result from other studies on similar data. This may be because the user cost term used is an imperfect or noisy measure of the true level that will give a bias towards zero (potential *attenuation bias*). Likewise, other studies indicate that a higher level of aggregation

implies that the referenced point estimates in the text may be different from the most recently used ones.

[7]These analyzes differ from the present i.a. by a slightly different industry or nest structure as the preferred specification, so a one-to-one comparison cannot be made immediately.

may also result in a downward bias in the estimates (potential *aggregation bias*). [8] Investigating whether these types of biases are the explanation behind the many zero-elasticities is interesting to investigate further, but is for now left to future work. A long run elasticity of 0 has the implication that production in an industry in the long run can only be shifted if the building capital moves. As the building capital is naturally a sluggish variable (even on annual frequency), this is not a good model feature in the short run. Therefore, low substitution elasticities on buildings must be supplemented with capacity utilization on building capital in the model (i.e. temporary deviations from the isoquant are allowed). One exception is for the extraction industry, where we find an elasticity significantly greater than 1. This in itself is not necessarily strange, as buildings in this industry firstly make up a significantly larger share of the total factor remuneration and secondly may have a completely different function than e.g. private services. The extraction industry also differs in that the relative factor remuneration for building capital has been steadily increasing during the estimation period (in contrast, the factor remuneration for buildings has been fairly constant in the manufacturing industry and declining from the early 1980s in the service industry). However, there is some noise and signs of a possible structural break in data in the extraction industry around 1980. Excluding the first part of the period gives a lower, but still relatively high elasticity, which is , however, significantly different from 0 and 1 at a 10% significance level .

In the KLB (R) nest, we also find zero elasticities for several industries. As R is a weighting of energy and material inputs, this is consistent with estimates using the same method, but with a KELM nest structure (see Kronborg et al., 2019). Here, we found that energy and materials typically had relatively low or no significant price substitution (similar to ADAM). Finally, its should be noted that the assumptions of short run dynamics may be significant for the estimated elasticity of materials: We have previously estimated the models in static form, i.e. where the economy is also implicitly assumed to be on the isoquant in the short run, similar to Thomsen. This specification generally provides higher substitution elasticities on material inputs. It should be noted that the nest structure here was slightly different (KELM, i.e. energy and materials were split). In the construction and manufacturing industry as well as in the energy industry (although to a lesser extent in the latter), we find positive and moderate substitution elasticities in the R nest, which is a robust result across model specifications and regarding the estimation period.

The processes of relative augmenting technological development between factor inputs are shown in the figures in Appendix B.[9] Technological development has generally been aimed at increasing the efficiency of labor rather than capital, which is in line with an interpretation where labor is the scarce factor (Acemoglu, 2002). This interpretation fits particularly well with the manufacturing industry, where the growth rate of the relative augmentation technology between

[8] An example is Chirinko and Mallick (2017), who find significantly lower elasticity on aggregated US data compared to a weighted average of the elasticity for 35 different industries.

[9] The figures in Appendix B are also the standard output from the CESKalman R package.

capital and labor has been largely constant throughout the period and aimed at improving the efficiency of labor. This also applies to the service industry in the first part of the time period, but from 1990 onwards, on the other hand, relative technological development has been largely constant and after the financial crisis has been aimed at improving the efficiency of capital. A similar pattern also applies in the construction industry. The other industries generally support labor-saving technological advances in the long run, but with significant fluctuations in the medium run (5-10 years), which underlines the importance of applying a flexible and time-varying specification of technological development. The relative augmented technological development between KL and B has been aimed at improving the efficiency of the KL nest across almost all industries. However, there are two exceptions. The first is the extraction industry, where technological development has been aimed at improving the efficiency of B. This is in line with the fact that this industry also has a significantly different elasticity between KL and B compared to the other industries. Secondly, the relative augmentation of technological development in the service industry has been aimed at improving the efficiency of KL until 1990 and subsequently at improving the efficiency of buildings. This is consistent with declining technological improvements that increase the efficiency of labor as observed in the substitution between K and L in the service industry. Finally, the relative augmenting technological development between KLB and R has been directed towards KLB in virtually all industries and often with a near constant growth rate. The only exceptions, however, are in the extraction industry, where technological development is directed towards R after the financial crisis in 2008, and the energy industry, where the relative augmentation technology is largely constant in the long run, but with significant fluctuations in the medium run.

# 5 Robustness

This section looks at the robustness of the estimated elasticities. Since the price of capital (i.e. the user cost) is the variable that depends most on the assumptions made, we first examine the effects of varying the user cost term by using another measure of the inflation expectations (Table 2) and another interest rate (Table 3). The robustness is then analyzed for a shorter estimation period (Table 4) and finally for the degree of smoothing for technological changes, i.e. values of $\lambda$ (see Table 12-18 in Appendix D).

We first look at an alternative formation of expectations to the rate of price increase of the relevant investment price in the user cost term. As mentioned, static inflation expectations give rise to a real interest rate that is very volatile at the beginning of the estimation period and which is therefore difficult to use for estimation purposes. An alternative to using the HP filter could therefore be to use adaptive expectations for the inflation rate. Table 2 shows the results with an adaptive expectation which is "updated" by a factor of 0.2 of the last period's inflation

Tabel 2: Robustness: Elasticities when using alternative inflation expectations.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| Agriculture | 0.01<br>(−0.13;0.19) | 0.00<br>(−0.07;0.04) | 0.00<br>(−0.08;0.03) |
| Construction | 0.03<br>(−0.22;0.40) | 0.00<br>(−0.27;0.23) | 0.37<br>(0.01;0.88) |
| Energy | 0.05<br>(−0.19;0.23) | 0.00<br>(−0.21;0.13) | 0.10<br>(−0.06;0.19) |
| Extraction | 0.31<br>(0.13;0.35) | 0.00<br>(−0.60;0.48) | 0.00<br>(−0.22;0.31) |
| Manufacturing | 0.23<br>(−0.04;0.39) | 0.00<br>(−0.14;0.14) | 0.51<br>(0.15;0.72) |
| Sea transport | 0.10<br>(−0.06;0.16) | 0.00<br>(−0.77;0.39) | 0.00<br>(−0.14;0.08) |
| Services | 0.46<br>(0.12;0.96) | 0.00<br>(−0.14;0.19) | 0.00<br>(−0.34;0.38) |

Note: Estimates are shown by industry and nest using adaptive inflation expectations. 5% and 95% percentiles in parentheses.

and with a weight of 0.8 on the last period's inflation expectations. In most industries and nests, inflation expectations mean relatively little for the estimates (usually only at the second decimal). For example, we still find the 0 elasticities for buildings in the preferred model specification. The elasticity of building capital in the extraction industry is the least robust. Like the other industries, it does not have a positive, significant elasticity using adaptive inflation expectations, although it is conceivable that there is a data break in the early 1980s, particularly in the extraction industry, make the estimates sensitive to the specification of user cost. Finally, adaptive inflation expectations cause a halving of the elasticity between K and L in the manufacturing industry, bringing it close to the estimate in ADAM.

The relevant interest rate in our preferred model is the same interest rate as used as in MAKRO, i.e. the average bond yield (IWBZ). As an alternative to this, we estimate the elasticities, where the banks' average lending rate (IWLO) is used instead. The two interest rate terms follow each other relatively closely, but there are differences, primarily at the beginning of the estimation period. The elasticities are shown in Table 3. As with the alternative inflation expectations, there is a limited effect of applying a different interest rate for most of the estimates. The service industry is the most notable, because no positive CES elasticity can be found. This underlines that price substitution here is generally weakly determined and associated with considerable uncertainty. Like the result from our preferred specification for the nest with bulding capital, we find that the extraction industry is the only one with a significant positive elasticity, and now with a point estimate of 0.75.

As mentioned above, some of the graphs in Appendix B (and perhaps particularly the extraction industry) indicate that there may be a structural break in the data series, including a shift to lower

Tabel 3: Robustness: Elasticities when using alternative interest rate variable.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| Agriculture | 0.11<br>(−0.16;0.36) | 0.00<br>(−0.10;0.05) | 0.00<br>(−0.12;0.04) |
| Construction | 0.06<br>(−0.53;0.38) | 0.00<br>(−0.17;0.15) | 0.34<br>(0.02;0.88) |
| Energy | 0.06<br>(−0.24;0.23) | 0.00<br>(−0.25;0.22) | 0.10<br>(−0.12;0.15) |
| Extraction | 0.19<br>(−0.01;0.24) | 0.75<br>(0.59;1.04) | 0.00<br>(−0.32;0.22) |
| Manufacturing | 0.37<br>(0.07;0.49) | 0.00<br>(−0.14;0.11) | 0.57<br>(0.24;0.87) |
| Sea transport | 0.11<br>(−0.13;0.24) | 0.00<br>(−0.76;0.47) | 0.00<br>(−0.23;0.12) |
| Services | 0.00<br>(−0.90;0.27) | 0.06<br>(−0.06;0.14) | 0.00<br>(−0.35;0.39) |

Note: Estimates are shown by industry and nest using IWLO as interest rate variable. 5% and 95% percentiles in parentheses.

volatility in the user cost terms ("great moderation"). Table 4 therefore shows the point estimates for the elasticities if the model is estimated from 1983 instead of the full period. First, it can be stated that the zero (and "almost zero") elasticities from our preferred specification are still found at the shorter estimation period. This indicates that the low elasticities are not solely due to the relatively volatile user cost terms at the beginning of the estimation period. The result in the construction and manufacturing industries, where we found substitution in relation to resources (R), is relatively robust compared to the shorter estimation period. The second main result that the extraction industry has a higher elasticity in relation to buildings is also found, although the point estimate is below 1. Again, the estimated elasticity between K and L in the service industry is low in this case. Most notable is the elasticity between K and L in the energy industry, rising from 0.04 using the full sample period to 1.67 with data starting from 1983. However, this fits well with the immediate impression from the data series in Figure 8, which shows that the relative prices and quantities follow closely from 1980 onwards.

Finally, we examine the robustness of our "identifying assumption", i.e. sluggish technological development. In addition to the allowable values in our main analysis, the limit values for $\lambda$ here are extended to be between 10 and 10,000. The results of the elasticities as a function of $\lambda$ are shown in Table 12-18 in Appendix D. We generally observe that the low elasticities are relatively unaffected by the signal-noise ratio. However, there is a considerable variation in the elasticity in nests such as (KLB)R in the construction industry, where they are ranging from 0.19 when $\lambda = 10$ to 0.93 when $\lambda = 10.000$. The elasticity is typically increasing in the value of $\lambda$. This is in line with the fact that when the process of technology is flexible, most of the variation in relative amounts is attributed to technological development rather than shifts in the relative prices. A similar pattern

Tabel 4: Robustness: Elasticities from estimation on sample starting in 1983.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| Agriculture | 0.00 | 0.00 | 0.00 |
| | (−0.27;0.31) | (−0.11;0.04) | (−0.14;0.04) |
| Construction | 0.06 | 0.00 | 0.45 |
| | (−1.05;0.28) | (−0.36;0.10) | (0.03;0.75) |
| Energy | 1.67 | 0.03 | 0.00 |
| | (1.31;3.22) | (−0.11;0.24) | (−0.28;−0.03) |
| Extraction | 0.14 | 0.55 | 0.00 |
| | (−0.59;0.28) | (0.37;0.90) | (−0.81;0.55) |
| Manufacturing | 0.31 | 0.00 | 0.30 |
| | (−0.11;0.51) | (−0.17;0.05) | (−0.08;0.46) |
| Sea transport | 0.00 | 0.00 | 0.00 |
| | (−0.34;0.21) | (−0.99;0.15) | (−0.28;0.12) |
| Services | 0.05 | 0.00 | 0.00 |
| | (−1.76;1.08) | (−0.13;0.12) | (−0.68;0.29) |

Note: Estimates are shown by industry and nest when using data from 1983 onwards. 5% and 95% percentiles in parentheses.

applies in the nests with moderate KL elasticities, e.g. manufacturing increasing from 0.08 to 0.42. In addition, most of the different values of $\lambda$ result in a well-specified model, whereby the choice of the optimal value is the one that maximizes likelihood. The best model in terms of data fit is rarely represented by a very high degree of smoothing.

# 6 Summary

The production side of MAKRO is characterized by a CES production function, further divided into a nested structure. We estimate the substitution elasticities between the factors of production: machinery capital, labor, building capital and resources (i.e. energy and material input). The CES elasticities in the production functions are estimated for all MAKRO's private industries (except housing), i.e. agriculture, construction, energy, extraction, manufacturing, sea transportation and services. As our preferred model specification, we estimate the CES elasticities based on the full period 1967-2017. The model is estimated on error correction form to allow some production factors to slowly adjust to a desired level, e.g. due to adjustment costs. Technological changes are allowed to be time varying, but the restriction is imposed that it must be a slow moving process. The approach results in well-specified models, based on a number of econometric tests as well as point estimates reminiscent of similar studies on Danish data. However, the CES elasticities are determined with some uncertainty. In several industries and nests, there are generally elasticities close to or equal to 0, which may be due to a downward bias due to noisy user cost expressions or a so-called aggregation bias. In future work, it will be interesting to investigate this further.

# Litteratur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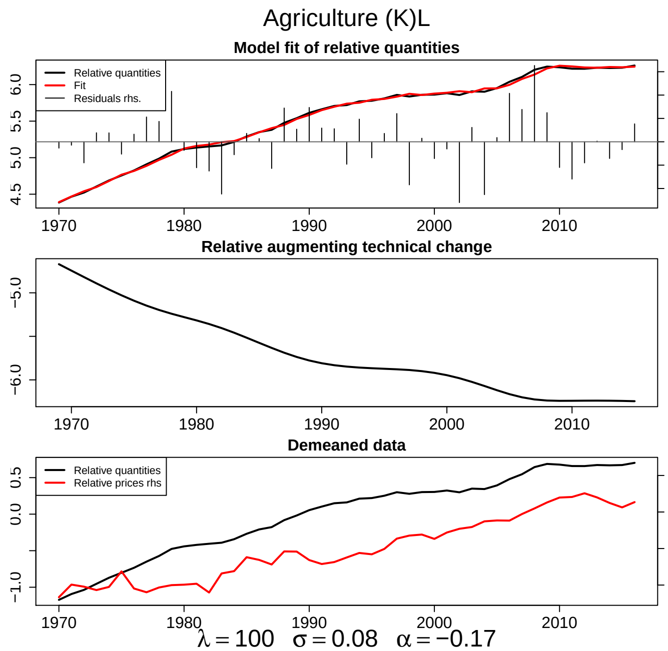
# A User cost

In the calculation of user cost, a distinction is made between machinery and building capital and a user cost expression is calculated for both types of capital. User cost of capital, $p^K_{k,sp,t}$, is derived from the firm's maximization problem, where $k$ is the type of capital, $sp$ is the industry and $t$ is the time period. The term is given by:

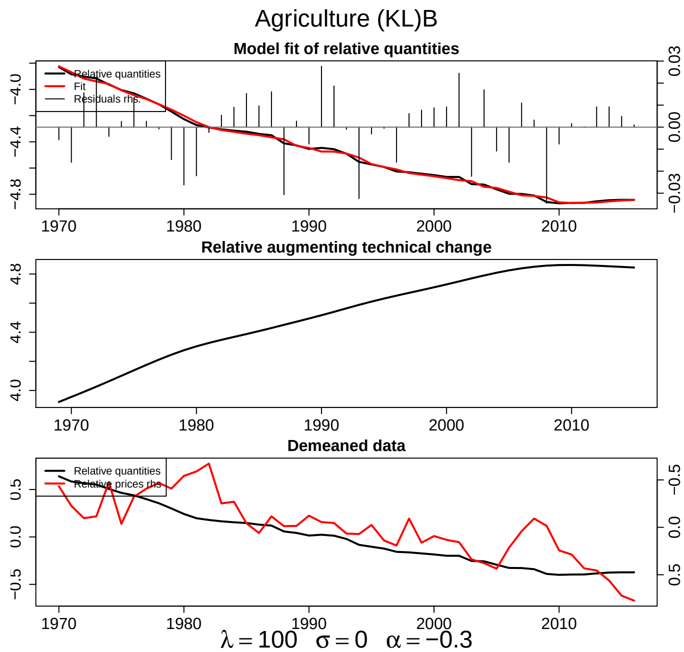
$$
\begin{gathered}
P^K_{k,sp,t} = \tfrac{1}{1-t_t^{Corp}} \cdot p^I_{k,sp,t-1} \cdot \\
[\left(1 - t_t^{Corp} \cdot \tfrac{x^{TaxDepr}_{k,t}}{x_t^{FirmDisc} + r^{TaxDepr}_{k,t}}\right)\left(x_t^{FirmDisc} + x^{DeprStatic}_{k,sp,t} - \left(1 - x^{DeprStatic}_{k,sp,t}\right) gp^{IStatic}_{k,sp,t}\right) \\
+ \left(1 - t_t^{Corp}\right) t^K_{t,sp,t} \left(1 + gp^{IStatic}_{k,sp,t}\right) \\
- t_t^{Corp} \cdot r_t^{Bond} \cdot x^{Debt}_{t-1}].
\end{gathered}
$$

$t_t^{corp}$ is the corporation tax rate, $p^I_{k,sp,t-1}$ is the investmentprice deflator by industry, $x^{TaxDepr}_{k,t}$ is the tax depreciation rate, $x_t^{FirmDisc}$ is the firms' discount rate, $x^{DeprStatic}_{k,sp,t}$ is the depreciation rate, $gp^{IStatic}_{k,sp,t}$ is the expected inflation rate for the investment price, $r_t^{Bond}$ is the bond yields og $x^{Debt}_{t-1}$ is the share of debt-financed investments. This is an expression of the static user cost, which does not include installation costs.

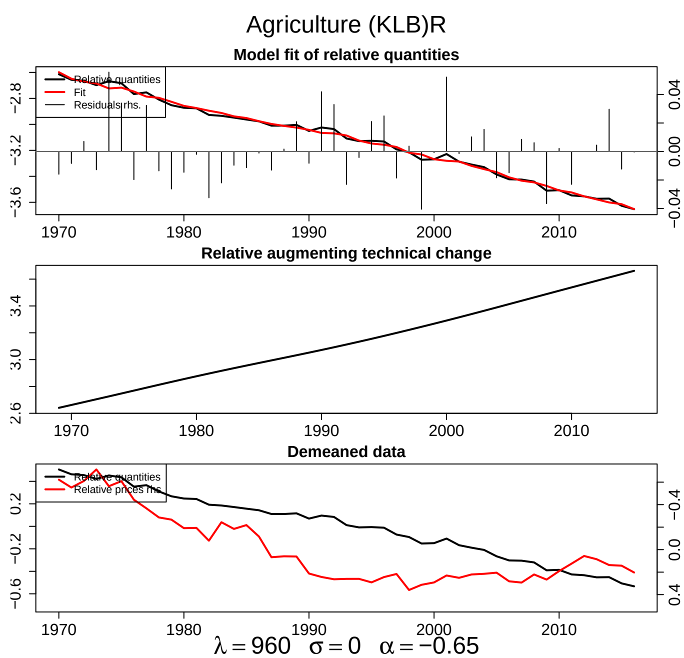
## B Graphic output from preferred model specification



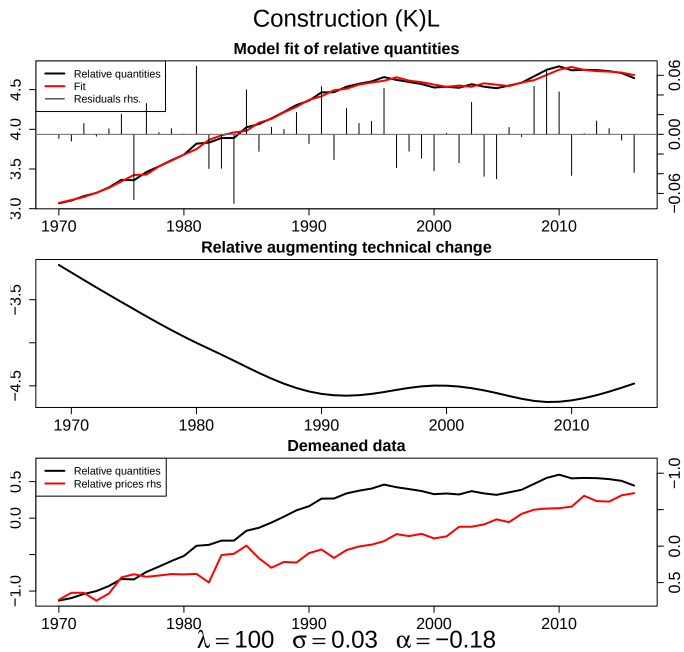

Figur 2: Agriculture: Graphic output from preferred model specification in nest K (L). The top graph shows the model's fit of the relative quantities as well as associated residuals. The middle graph shows relative augmentation technological progress, $log\,(\Gamma_{1t}/\Gamma_{2t})$. The bottom graph shows the relative quantities as well as relative prices, both minus their mean value. The prices are shown as the inverse, i.e. if they follow each other they are negatively correlated.



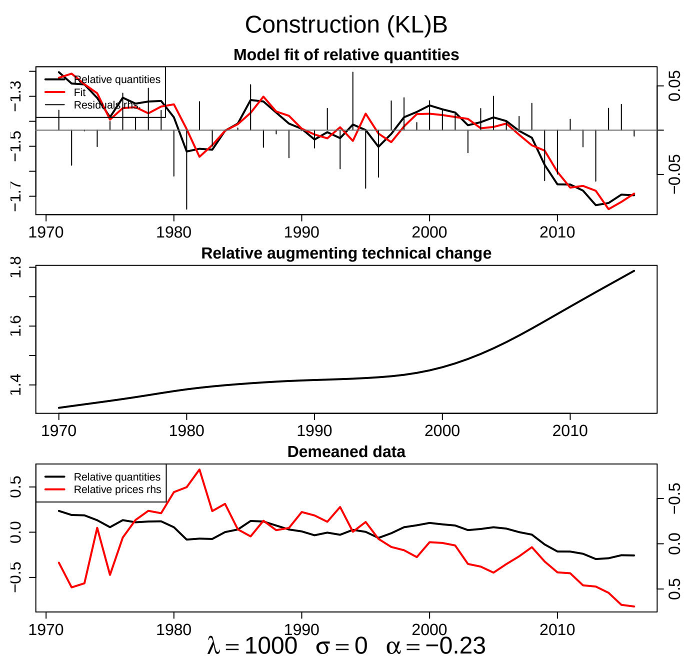

Figur 3: Agriculture: Graphic output from preferred model specification in nest KL (B). See Figure 2 for description.



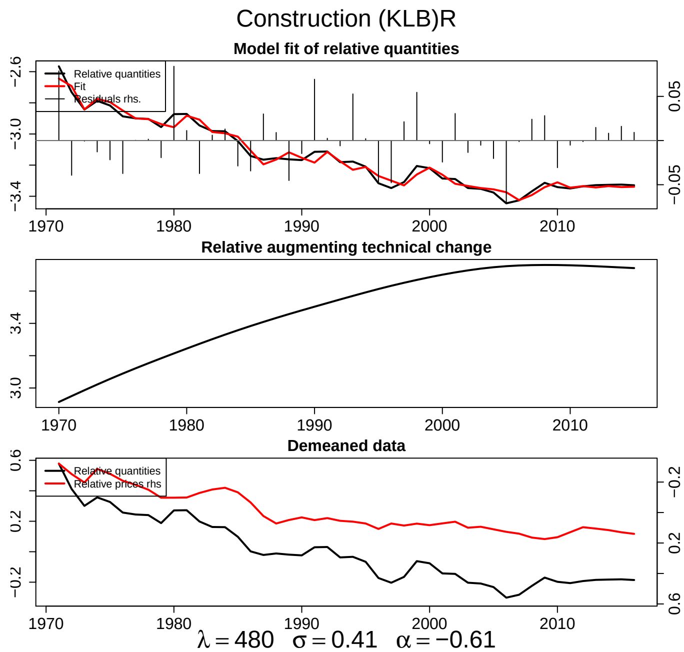

Figur 4: Agriculture: Graphic output from preferred model specification in nest KLB (R). See Figure 2 for description.



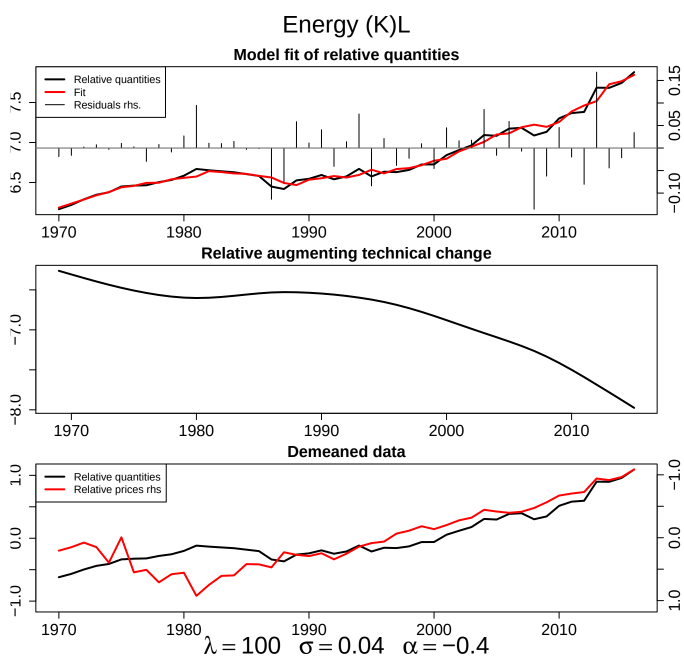

Figur 5: Construction: Graphic output from preferred model specification in nest K(L). See Figure 2 for description.



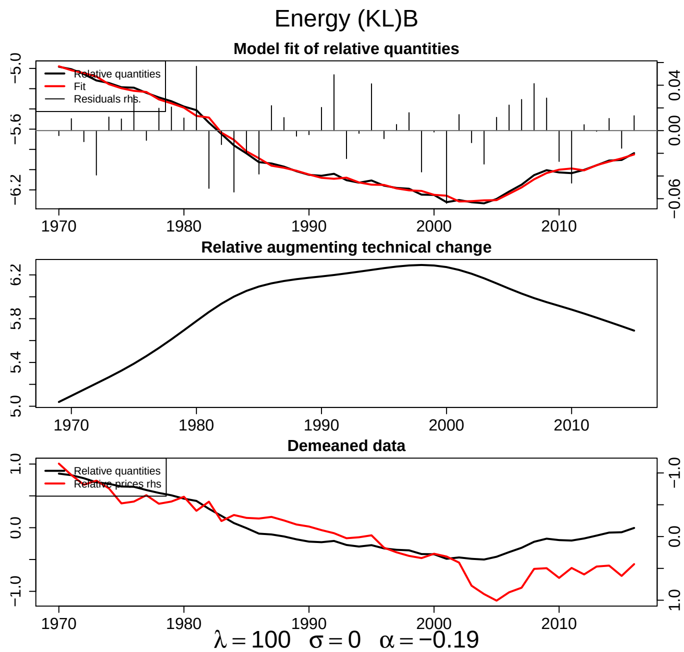

Figur 6: Construction: Graphic output from preferred model specification in nest KL(B). See Figure 2 for description.



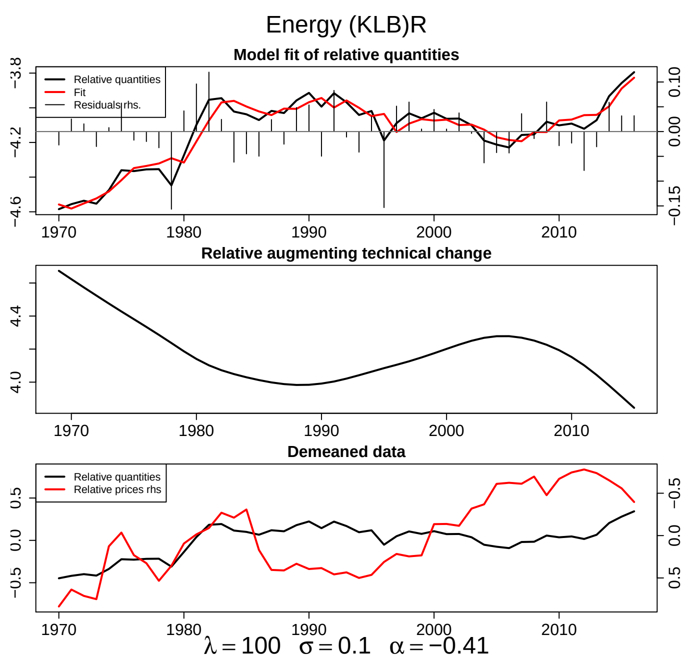

Figur 7: Construction: Graphic output from preferred model specification in nest KLB(R). See Figure 2 for description.



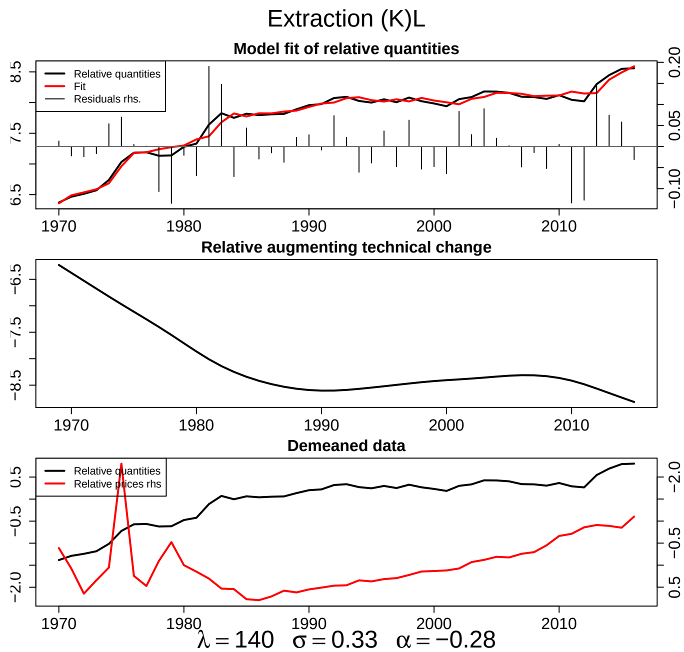

Figur 8: Energy: Graphic output from preferred model specification in nest K(L). See Figure 2 for description.



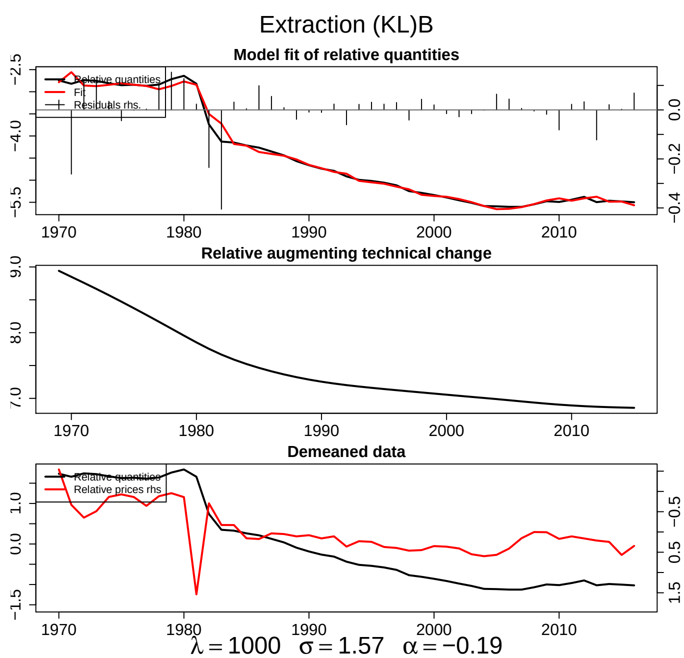

Figur 9: Energy: Graphic output from preferred model specification in nest KL(B). See Figure 2 for description.



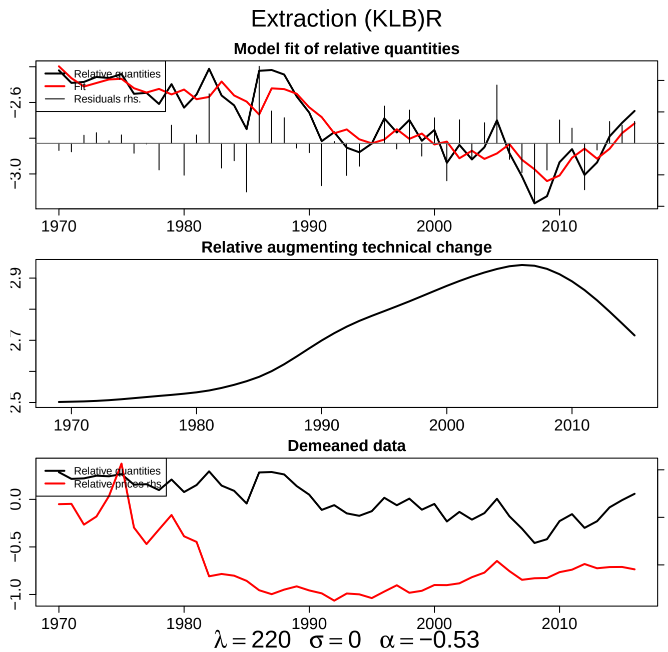

Figur 10: Energy: Graphic output from preferred model specification in nest KLB(R). See Figure 2 for description.



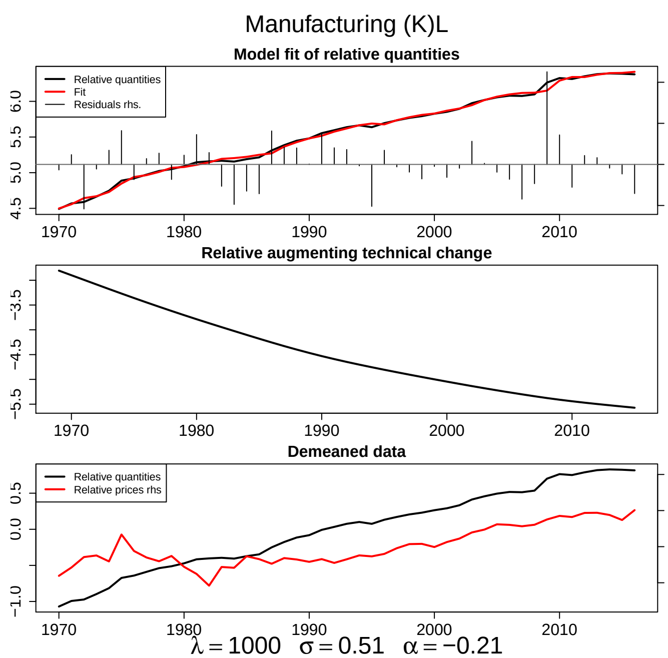

Figur 11: Extraction: Graphic output from preferred model specification in nest K(L). See Figure 2 for description.



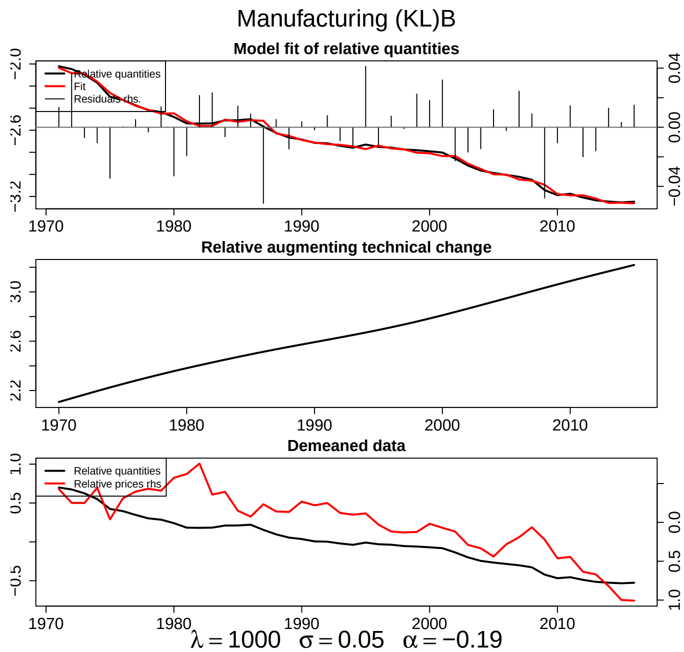

Figur 12: Extraction: Graphic output from preferred model specification in nest KL(B). See Figure 2 for description.



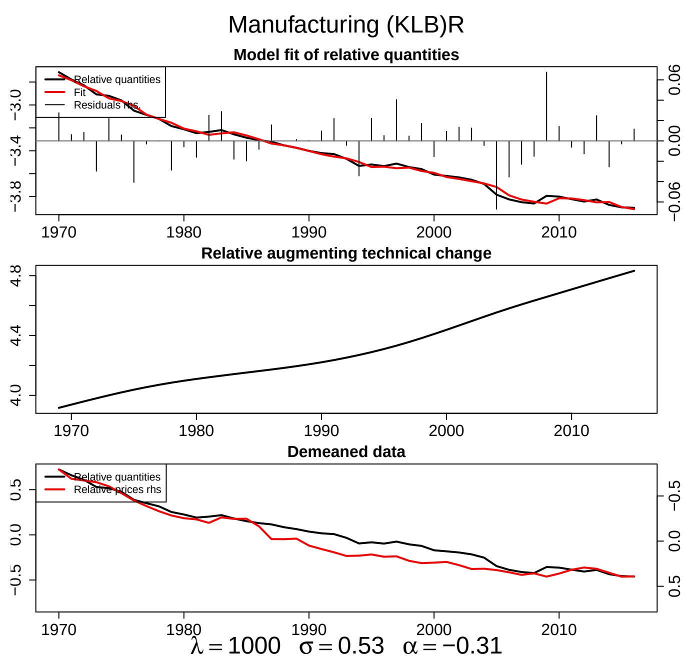

Figur 13: Extraction: Graphic output from preferred model specification in nest KLB(R). See Figure 2 for description.



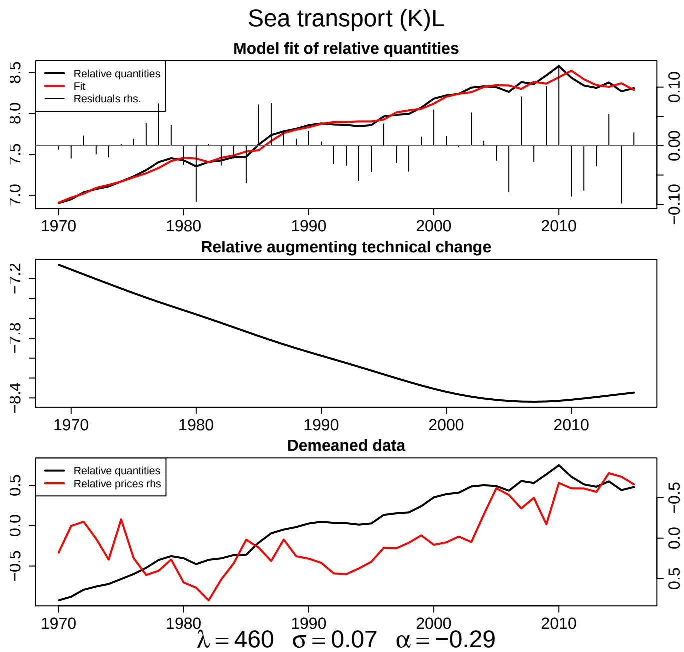

Figur 14: Manufacturing: Graphic output from preferred model specification in nest K(L). See Figure 2 for description.



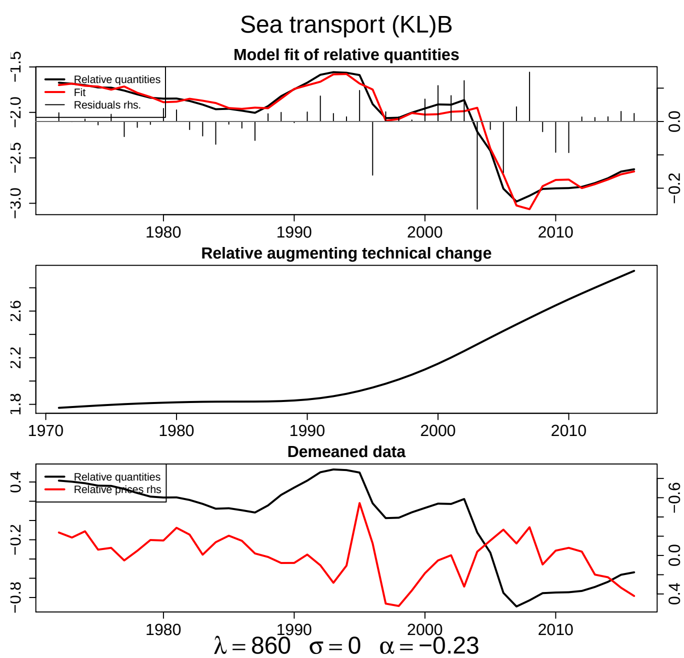

Figur 15: Manufacturing: Graphic output from preferred model specification in nest KL(B). See Figure 2 for description.



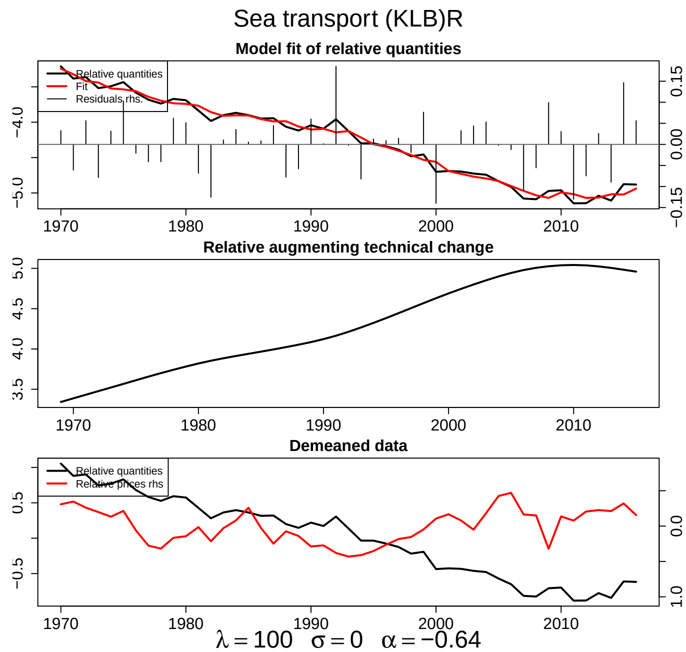

Figur 16: Manufacturing: Graphic output from preferred model specification in nest KLB(R). See Figure 2 for description.



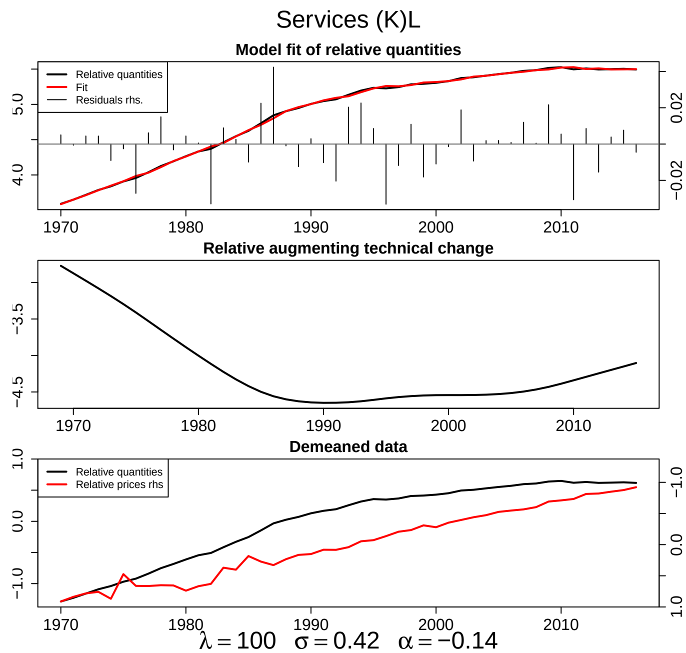

Figur 17: Sea transportation: Graphic output from preferred model specification in nest K(L). See Figure 2 for description.



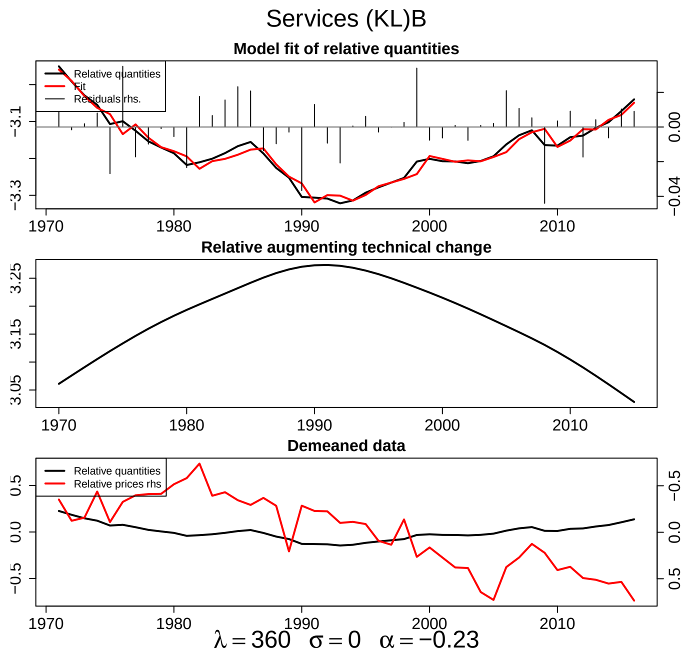

Figur 18: Sea transportation: Graphic output from preferred model specification in nest KL(B). See Figure 2 for description.



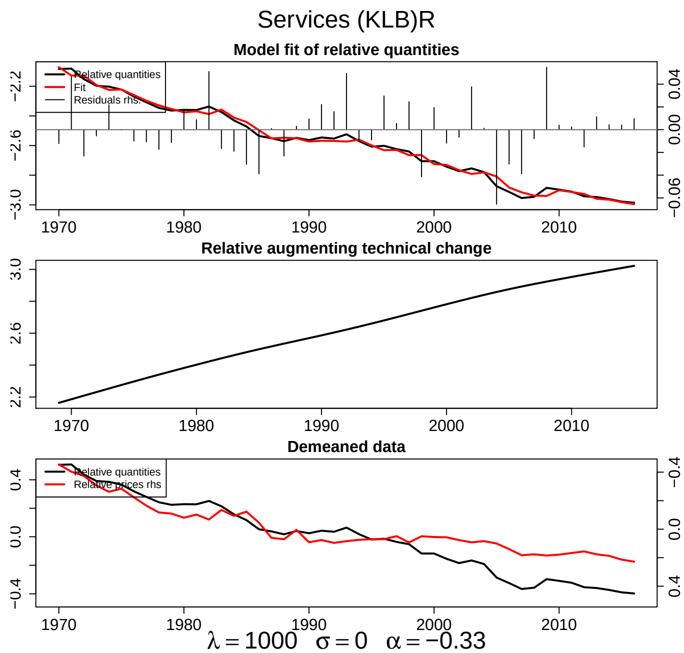

Figur 19: Sea transportation: Graphic output from preferred model specification in nest KLB(R). See Figure 2 for description.




Figur 20: Services: Graphic output from preferred model specification in nest K(L). See Figure 2 for description.




Figur 21: Services: Graphic output from preferred model specification in nest KL(B). See Figure 2 for description.




Figur 22: Services: Graphic output from preferred model specification in nest KLB(R). See Figure 2 for description.

# C Detailed tables for estimates (preferred model specification)

Tabel 5: Agriculture: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.08 | 0.00 | 0.00 |
| | $(-0.14;0.33)$ | $(-0.10;0.01)$ | $(-0.12;0.04)$ |
| $\alpha$ | $-0.17$ | $-0.30$ | $-0.65$ |
| | $(-0.53;-0.22)$ | $(-0.64;-0.32)$ | $(-0.93;-0.54)$ |
| nlags | 0 | 0 | 0 |
| Likelihood | 116.91 | 140.98 | 129.87 |
| $\lambda$ | 100.00 | 100.00 | 960.00 |
| Autocorrelation | [ 0.12 ] | [ 0.50 ] | [ 0.84 ] |
| Heteroskedasticity | [ 0.21 ] | [ 0.97 ] | [ 0.65 ] |
| Normality | [ 0.94 ] | [ 0.17 ] | [ 0.10 ] |
| NIS | 0.79 | 0.82 | 0.87 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

Tabel 6: Construction: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.03 | 0.00 | 0.41 |
| | $(-0.47;0.42)$ | $(-0.22;0.15)$ | $(0.05;0.95)$ |
| $\alpha$ | $-0.18$ | $-0.23$ | $-0.61$ |
| | $(-0.42;-0.18)$ | $(-0.51;-0.19)$ | $(-0.89;-0.49)$ |
| nlags | 0 | 1 | 1 |
| Likelihood | 103.81 | 98.21 | 101.44 |
| $\lambda$ | 100.00 | 1000.00 | 480.00 |
| Autocorrelation | [ 0.57 ] | [ 0.75 ] | [ 0.77 ] |
| Heteroskedasticity | [ 0.74 ] | [ 0.83 ] | [ 0.79 ] |
| Normality | [ 0.86 ] | [ 0.13 ] | [ 0.16 ] |
| NIS | 0.78 | 0.82 | 0.78 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

Tabel 7: Energy: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.04 <br> $(-0.31;0.15)$ | 0.00 <br> $(-0.25;0.21)$ | 0.10 <br> $(-0.11;0.18)$ |
| $\alpha$ | $-0.40$ <br> $(-0.67;-0.38)$ | $-0.19$ <br> $(-0.34;-0.17)$ | $-0.41$ <br> $(-0.68;-0.36)$ |
| nlags | 0 | 0 | 0 |
| Likelihood | 82.73 | 112.80 | 80.05 |
| $\lambda$ | 100.00 | 100.00 | 100.00 |
| Autocorrelation | [ 0.35 ] | [ 0.74 ] | [ 0.66 ] |
| Heteroskedasticity | [ 0.23 ] | [ 0.35 ] | [ 0.60 ] |
| Normality | [ 0.05 ] | [ 0.51 ] | [ 0.08 ] |
| NIS | 0.81 | 0.80 | 0.81 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

Tabel 8: Extraction: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.33 <br> $(0.05;0.38)$ | 1.57 <br> $(0.85;2.12)$ | 0.00 <br> $(-0.32;0.26)$ |
| $\alpha$ | $-0.28$ <br> $(-0.54;-0.27)$ | $-0.19$ <br> $(-0.33;-0.16)$ | $-0.53$ <br> $(-0.90;-0.49)$ |
| nlags | 0 | 0 | 0 |
| Likelihood | 69.51 | 59.28 | 61.89 |
| $\lambda$ | 140.00 | 1000.00 | 220.00 |
| Autocorrelation | [ 0.29 ] | [ 0.14 ] | [ 0.97 ] |
| Heteroskedasticity | [ 0.59 ] | [ 0.21 ] | [ 0.26 ] |
| Normality | [ 0.36 ] | [ 0.00 ] | [ 0.74 ] |
| NIS | 0.82 | 0.88 | 0.84 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

Tabel 9: Manufacturing: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.51 (0.10;0.70) | 0.05 (−0.18;0.25) | 0.53 (0.21;0.69) |
| $\alpha$ | −0.21 (−0.49;−0.19) | −0.19 (−0.35;−0.15) | −0.31 (−0.63;−0.27) |
| nlags | 0 | 1 | 0 |
| Likelihood | 112.98 | 118.18 | 125.15 |
| $\lambda$ | 1000.00 | 1000.00 | 1000.00 |
| Autocorrelation | [ 0.28 ] | [ 0.76 ] | [ 0.45 ] |
| Heteroskedasticity | [ 0.56 ] | [ 0.83 ] | [ 0.63 ] |
| Normality | [ 0.00 ] | [ 0.41 ] | [ 0.18 ] |
| NIS | 0.87 | 0.81 | 0.87 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

Tabel 10: Sea transportation: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.07 (−0.15;0.21) | 0.00 (−0.95;0.33) | 0.00 (−0.20;0.12) |
| $\alpha$ | −0.29 (−0.66;−0.31) | −0.23 (−0.44;−0.19) | −0.64 (−0.88;−0.50) |
| nlags | 0 | 2 | 0 |
| Likelihood | 86.30 | 58.81 | 73.32 |
| $\lambda$ | 460.00 | 860.00 | 100.00 |
| Autocorrelation | [ 0.59 ] | [ 0.98 ] | [ 0.67 ] |
| Heteroskedasticity | [ 0.04 ] | [ 0.14 ] | [ 0.31 ] |
| Normality | [ 0.70 ] | [ 0.00 ] | [ 0.97 ] |
| NIS | 0.85 | 0.75 | 0.80 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

Tabel 11: Services: Estimated results in a ((KL) B) R nest structure.

| | (K)L | (KL)B | (KLB)R |
|---|---|---|---|
| $\sigma$ | 0.42 | 0.00 | 0.00 |
| | $(-0.08;0.84)$ | $(-0.15;0.12)$ | $(-0.29;0.41)$ |
| $\alpha$ | $-0.14$ | $-0.23$ | $-0.33$ |
| | $(-0.22;-0.10)$ | $(-0.36;-0.18)$ | $(-0.73;-0.29)$ |
| nlags | 0 | 1 | 0 |
| Likelihood | 137.07 | 129.93 | 125.18 |
| $\lambda$ | 100.00 | 360.00 | 1000.00 |
| Autocorrelation | [ 1.00 ] | [ 0.13 ] | [ 0.55 ] |
| Heteroskedasticity | [ 0.13 ] | [ 0.92 ] | [ 0.84 ] |
| Normality | [ 0.70 ] | [ 0.25 ] | [ 0.92 ] |
| NIS | 0.79 | 0.80 | 0.89 |

Note: The expressions in parentheses are lower and upper critical values at a 10% significance level, respectively. P-values for misspecification tests are in square brackets. The critical values for NIS are (0.68; 1.37) at a 10% significance level.

# D Importance of smoothing

Tabel 12: Agriculture: Sensitivity to different degrees of smoothness for technological development.

| | | K(L) | | | | KL(B) | | | | KLB(R) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.01 | 0.68 | 0.53 | 119.03 | 0.00 | 0.73 | 0.83 | 141.61 | 0.09 | 0.78 | 0.28 | 125.56 |
| 50 | 0.03 | 0.76 | 0.29 | 117.41 | 0.00 | 0.80 | 0.61 | 141.10 | 0.08 | 0.79 | 0.59 | 126.58 |
| 100 | 0.08 | 0.79 | 0.12 | 116.91 | 0.00 | 0.82 | 0.50 | 140.98 | 0.07 | 0.80 | 0.76 | 126.55 |
| 500 | 0.01 | 0.75 | 0.05 | 103.11 | 0.00 | 0.90 | 0.40 | 140.64 | 0.02 | 0.88 | 0.95 | 126.16 |
| 1,000 | 0.03 | 0.75 | 0.05 | 102.16 | 0.00 | 0.91 | 0.38 | 140.47 | 0.00 | 0.91 | 0.84 | 129.87 |
| 10,000 | 0.00 | 0.80 | 0.80 | 104.03 | 0.00 | 0.94 | 0.39 | 139.83 | 0.00 | 0.92 | 0.82 | 129.20 |

Tabel 13: Construction: Sensitivity to different degrees of smoothness for technological development.

| | | K(L) | | | | KL(B) | | | | KLB(R) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.12 | 0.64 | 0.02 | 88.81 | 0.00 | 0.77 | 0.41 | 99.64 | 0.19 | 0.80 | 0.28 | 99.76 |
| 50 | 0.02 | 0.76 | 0.29 | 104.32 | 0.00 | 0.81 | 0.12 | 99.75 | 0.35 | 0.77 | 0.78 | 100.12 |
| 100 | 0.03 | 0.78 | 0.57 | 103.81 | 0.00 | 0.80 | 0.41 | 97.61 | 0.36 | 0.78 | 0.79 | 100.77 |
| 500 | 0.08 | 0.82 | 0.71 | 102.92 | 0.00 | 0.81 | 0.64 | 98.16 | 0.42 | 0.78 | 0.78 | 101.44 |
| 1,000 | 0.10 | 0.82 | 0.53 | 102.35 | 0.00 | 0.82 | 0.75 | 98.21 | 0.51 | 0.78 | 0.81 | 101.30 |
| 10,000 | 10.16 | 0.87 | 0.25 | 100.78 | 0.00 | 0.83 | 0.99 | 98.05 | 0.93 | 0.81 | 0.73 | 100.24 |

Tabel 14: Energy: Sensitivity to different degrees of smoothness for technological development.

| | K(L) | | | | KL(B) | | | | KLB(R) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.11 | 0.73 | 0.20 | 82.88 | 0.12 | 0.68 | 0.21 | 112.27 | 0.17 | 0.75 | 0.89 | 79.20 |
| 50 | 0.03 | 0.79 | 0.40 | 82.83 | 0.00 | 0.78 | 0.91 | 113.34 | 0.12 | 0.80 | 0.65 | 80.04 |
| 100 | 0.04 | 0.81 | 0.35 | 82.73 | 0.00 | 0.80 | 0.74 | 112.80 | 0.10 | 0.81 | 0.66 | 80.05 |
| 500 | 0.13 | 0.84 | 0.29 | 82.18 | 0.00 | 0.83 | 0.37 | 111.52 | 0.10 | 0.81 | 0.72 | 78.98 |
| 1,000 | 0.26 | 0.85 | 0.28 | 81.93 | 0.00 | 0.82 | 0.29 | 110.62 | 0.15 | 0.82 | 0.65 | 78.30 |
| 10,000 | 1.77 | 0.89 | 0.28 | 81.33 | 1.77 | 0.86 | 0.15 | 106.47 | 0.46 | 0.88 | 0.43 | 77.27 |

Tabel 15: Extraction: Sensitivity to different degrees of smoothness for technological development.

| | K(L) | | | | KL(B) | | | | KLB(R) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.23 | 0.76 | 0.88 | 68.11 | 0.85 | 0.78 | 0.56 | 54.52 | 0.10 | 0.78 | 0.65 | 59.60 |
| 50 | 0.29 | 0.81 | 0.40 | 69.22 | 1.21 | 0.84 | 0.27 | 56.64 | 0.04 | 0.82 | 0.91 | 61.22 |
| 100 | 0.32 | 0.82 | 0.31 | 69.47 | 1.34 | 0.86 | 0.22 | 57.43 | 0.01 | 0.83 | 1.00 | 61.63 |
| 500 | 0.47 | 0.83 | 0.26 | 69.20 | 1.53 | 0.88 | 0.15 | 58.89 | 0.00 | 0.85 | 0.97 | 61.52 |
| 1,000 | 0.76 | 0.85 | 0.27 | 68.95 | 1.57 | 0.88 | 0.14 | 59.28 | 0.00 | 0.86 | 0.95 | 61.21 |
| 10,000 | 2.99 | 0.89 | 0.24 | 68.59 | 1.87 | 0.88 | 0.14 | 59.62 | 0.03 | 0.88 | 0.65 | 61.52 |

Tabel 16: Manufacturing: Sensitivity to different degrees of smoothness for technological development.

| | K(L) | | | | KL(B) | | | | KLB(R) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.08 | 0.73 | 0.74 | 110.84 | 0.00 | 0.70 | 0.30 | 125.43 | 0.29 | 0.76 | 0.98 | 123.35 |
| 50 | 0.30 | 0.82 | 0.45 | 111.31 | 0.00 | 0.74 | 0.15 | 117.58 | 0.44 | 0.81 | 0.59 | 124.56 |
| 100 | 0.39 | 0.84 | 0.38 | 111.76 | 0.01 | 0.76 | 0.27 | 117.89 | 0.49 | 0.83 | 0.55 | 124.87 |
| 500 | 0.50 | 0.87 | 0.29 | 112.77 | 0.03 | 0.80 | 0.61 | 118.06 | 0.55 | 0.85 | 0.48 | 125.12 |
| 1,000 | 0.51 | 0.87 | 0.28 | 112.98 | 0.05 | 0.81 | 0.76 | 118.18 | 0.53 | 0.87 | 0.45 | 125.15 |
| 10,000 | 0.42 | 0.87 | 0.19 | 112.46 | 0.10 | 0.83 | 0.96 | 118.55 | 0.40 | 0.89 | 0.36 | 125.14 |

Tabel 17: Sea transportation: Sensitivity to different degrees of smoothness for technological development.

| | K(L) | | | | KL(B) | | | | KLB(R) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.00 | 0.74 | 0.98 | 87.70 | 0.00 | 0.69 | 0.18 | 64.94 | 0.00 | 0.80 | 0.58 | 72.23 |
| 50 | 0.02 | 0.82 | 0.75 | 85.45 | 0.00 | 0.70 | 0.48 | 58.56 | 0.00 | 0.81 | 0.66 | 73.53 |
| 100 | 0.05 | 0.84 | 0.67 | 85.86 | 0.00 | 0.72 | 0.64 | 58.63 | 0.00 | 0.80 | 0.67 | 73.32 |
| 500 | 0.07 | 0.85 | 0.58 | 86.30 | 0.00 | 0.75 | 0.96 | 58.79 | 0.00 | 0.83 | 0.82 | 72.07 |
| 1,000 | 0.09 | 0.85 | 0.62 | 86.23 | 0.00 | 0.75 | 0.96 | 58.81 | 0.00 | 0.85 | 0.94 | 71.59 |
| 10,000 | 0.01 | 0.88 | 0.61 | 83.60 | 0.00 | 0.77 | 0.72 | 58.23 | 0.00 | 0.90 | 0.66 | 70.87 |

Tabel 18: Services: Sensitivity to different degrees of smoothness for technological development.

| | K(L) | | | | KL(B) | | | | KLB(R) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV | $\sigma$ | NIS | Auto | LV |
| 10 | 0.06 | 0.73 | 0.64 | 137.40 | 0.00 | 0.70 | 0.94 | 131.28 | 0.00 | 0.78 | 0.68 | 122.43 |
| 50 | 0.29 | 0.78 | 0.85 | 137.40 | 0.00 | 0.77 | 0.20 | 132.79 | 0.00 | 0.84 | 0.95 | 123.83 |
| 100 | 0.42 | 0.79 | 1.00 | 137.07 | 0.00 | 0.73 | 0.08 | 119.21 | 0.00 | 0.85 | 0.86 | 124.29 |
| 500 | 0.71 | 0.79 | 0.49 | 135.31 | 0.00 | 0.80 | 0.15 | 129.90 | 0.00 | 0.88 | 0.63 | 124.94 |
| 1,000 | 0.72 | 0.81 | 0.29 | 134.27 | 0.00 | 0.80 | 0.20 | 129.71 | 0.00 | 0.89 | 0.55 | 125.18 |
| 10,000 | 0.00 | 0.85 | 0.86 | 126.16 | 0.00 | 0.85 | 0.68 | 128.81 | 0.00 | 0.93 | 0.43 | 125.83 |